

# Aggregate Implications of Credit Market Imperfections

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## **Organization of the paper** (not this presentation):

1. Introduction
2. A Simple Model of Credit Market Imperfections: A Single Agent's Perspective
3. Partial Equilibrium Models
  - Homogenous Agents: Net Worth (Balance Sheet) Effect
  - Heterogeneous Agents: Distributional Implications
  - Heterogeneous Agents: Replacement Effects
4. General Equilibrium with Endogenous Saving: Capital Deepening vs. Net Worth Effects
5. General Equilibrium with Heterogeneous Projects
  - A Model with Pure Capital Projects: Endogenous Investment-Specific Technical Change:
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  - A Model with Private Benefits: Credit Cycles
  - A Model with Pure Capital and Consumption Projects:
    - Inefficient Recessions: Financial Accelerator
    - Inefficient Booms and Volatility
    - Hybrid Cases: Asymmetric Cycles & Intermittent Volatility
6. General Equil. with Hetero. Agents (and Capital): Patterns of International Capital Flows
7. General Equil. with Hetero. Agents (with Hetero. Projects): Patterns of International Trade
8. A Model of Polarization
9. Concluding Remarks

## What this paper does:

- By using *the same, simple* abstract model of credit market imperfections throughout,
- *synthesize* a diverse set of results within a *unified* framework.
- show how the credit market imperfections can be a key to understanding a wide range of aggregate phenomena, including:
  - Endogenous investment-specific technological changes
  - Development traps and Leapfrogging
  - Persistent recessions and recurrent boom-and-bust cycles
  - Reverse international capital flows
  - Rise and fall of Inequality across nations
  - New sources of comparative advantage and patterns of international trade
- with the hope of offering a coherent picture across many results that are *seemingly conflicting* and/or *seemingly unrelated*.

## Recurring themes:

- Properties of equilibrium often respond *non-monotonically* to parameter changes. For example,
  - Improving borrower net worth or credit market may first lead to a higher market rate of return and then to a lower market rate of return
  - Improving credit market may first lead to an increased volatility and then a reduced volatility.
  - Productivity improvement may first lead to a greater inequality and then a reduced inequality.

etc.

- Equilibrium and welfare consequences of the credit market imperfections are *rich and diverse* depending on the general equilibrium feedback mechanisms.

*What are the basic messages?*

**(To the outsider of the field):**

This is an exciting field, as credit market imperfections have such rich implications.

**(To the insider of the field):**

*Non-monotonicity, in particular, suggests*

- Drawing policy implications by comparing a model with credit market imperfections and a model without can be also dangerous, because the effects of improving the credit market could be very different from those of eliminating the credit market imperfections completely.
- The effects of imperfect credit markets could also be very different from the effects of no credit market.

*More generally,*

Some cautions for studying the equilibrium implications within *a narrow class or a particular family* of models and extrapolating from it.

“All happy families resemble one another. Each unhappy family is unhappy in its own way.”

Leo Tolstoy, *Anna Karenina*

***A Single Agent's Problem:*** serve as the building block in all the equilibrium models to come

**Two Periods:**  $t = 0$  and  $t = 1$

**A Single Agent (an Entrepreneur or a Firm):**

- is endowed with  $\omega < 1$  units of the input at period 0.
- consumes only at period 1.

**Two Means to Convert the Input into Consumption:**

- Run a **non-divisible project**, which converts one unit of the input in period 0 into  $R$  units in **Consumption** in period 1, by **borrowing**  $1-\omega$  at the market rate of return equal to  $r$ .
- **Lend**  $x \leq \omega$  units of the input in period 0 for  $rx$  units of consumption in period 1. (Or, **Storage** with the rate of return equal to  $r$ .)

**Agent's Utility = Consumption in period 1:**

$$\begin{aligned} U &= R - r(1-\omega) = R - r + r\omega, && \text{if borrow and run the project,} \\ U &= r\omega && \text{if lend (or put in storage).} \end{aligned}$$

***Profitability Constraint:*** The agent *is willing to* borrow and invest iff

$$(PC) \quad R \geq r$$

**Borrowing Constraint:** To borrow from the market, the agent must generate the market rate of return,  $r$ , per unit to the lenders, yet, *for a variety of reasons*, no more than a fraction,  $\lambda$ , of the project output can be used for this purpose. Thus, the agent *can* borrow and invest iff

$$(BC) \quad \lambda R \geq r(1-\omega).$$

If  $\lambda/(1-\omega) < r/R \leq 1$ , (PC) holds but not (BC).

- The profitable project fails to be financed, due to the borrowing constraint.
- **Necessary Condition:  $\lambda + \omega < 1$**
- A higher  $\omega$  (as well as a higher  $\lambda$ ) can alleviate the problem

### **Broad Interpretations of the Parameters:**

$\lambda$ : agency problems affecting credit transactions (may vary across projects or industries), institutional quality or the state of financial development (may vary across countries)

$\omega$ : entrepreneur's net worth, the firm's balance sheet, the borrower's credit-worthiness (may vary across agents).

*We now start endogenizing  $R$ ,  $r$ , and  $\omega$  (but not  $\lambda$ )*

## *Partial Equilibrium with Homogeneous Agents*

### **Two Departures:**

- **A Continuum of Homogeneous Agents with Unit Mass**
- A Project produces  $R$  units of **Capital**, used in the production of the Consumption Good,  $f(k) = F(k, \zeta)$ , where  $F(k, \zeta)$  is CRS but  $f(k)$  is subject to **Diminishing Returns**.  $\zeta$  is the hidden factors in fixed supply, owned by those who do not have access to the investment technologies.
- $k = Rn$  is Aggregate Supply of Capital;  $n$  is the number of agents running the project.

**Profitability Constraint (PC):**  $Rf'(k) \geq r$

**Borrowing Constraint (BC):**  $\lambda Rf'(k) \geq r(1-\omega)$ .

**Equilibrium Condition:**  $Rf'(k)/r = \text{Max}\{(1-\omega)/\lambda, 1\}$

If  $\lambda + \omega < 1$ ,  $Rf'(k) = r(1-\omega)/\lambda > r$ ;      **Under-Investment;**  
**Net Worth Effect;**  $\omega \uparrow \rightarrow k \uparrow$

If  $\lambda + \omega > 1$ ,  $Rf'(k) = r > r(1-\omega)/\lambda$ ;      **Optimal Investment;**  
**No Net Worth Effect.**



**Partial Equilibrium with Heterogeneous Agents:**  $\omega \sim G(\omega)$  with the same R.

If  $Rf'(k) > r$ ; Only those with  $\omega \geq \omega_c$  invest.

$$\rightarrow k = R[1 - G(\omega_c)] = R \left[ 1 - G \left( 1 - \frac{\lambda Rf'(k)}{r} \right) \right].$$

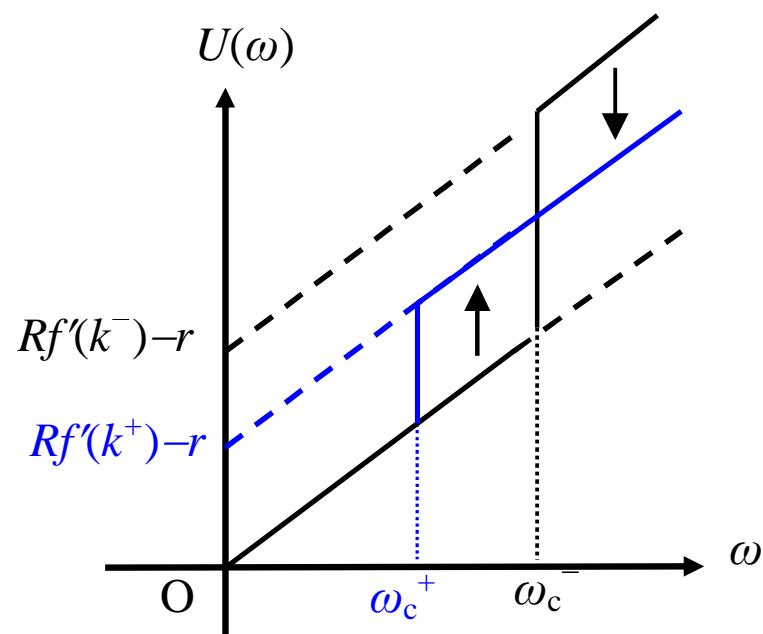
**Comparative Statics:**  $\lambda \uparrow \rightarrow \omega_c \downarrow, k \uparrow$

**Distributional Impacts of  $\lambda \uparrow$ :**

*The Middle Class (and those who own the hidden factors) gain; the Rich lose.*

Credit Market Imperfections as **Barriers to Entry**

$\rightarrow$  Political Economy Implications



**Partial Equilibrium with Heterogeneous Agents:  $(\omega, R) \sim G(\omega, R)$**

The investing agents must satisfy both

(PC)  $Rf'(k)/r \geq 1$

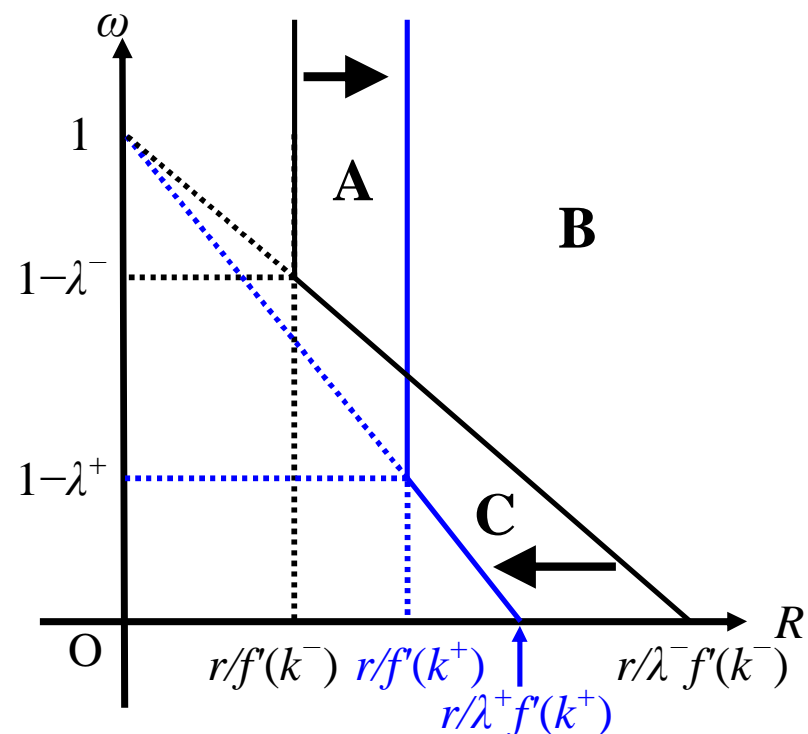
and

(BC)  $\omega \geq \omega_c(k) \equiv 1 - \lambda Rf'(k)/r$

$$\rightarrow k = \int_{\frac{r}{f'(k)}}^{\infty} R \left[ \int_{\omega_c(k)}^{\infty} g(\omega, R) d\omega \right] dR$$

**Composition Effects of Improved Credit Market**

The rich, but less productive agents in A replaced by the poor, but more productive agents in C.



Also, with a higher  $\lambda$ ,

- A fraction of the active firms that are credit-constrained first goes up and then goes down.
- Aggregate Investment may decline, as the credit shifts towards the more productive.

## A General Equilibrium Model with Endogenous Saving:

- Go back to the homogeneous case, where every (investing) agent has the same  $R$  and  $\omega$ .
  - Add some “savers”, with no access to the investment technology, who choose to maximize  $U^0 = V(C^0_0) + C^0_1$  subject to  $C^0_1 = r(\omega^0 - C^0_0)$ .
- Saving by the Savers:  $V'(\omega^0 - S^0(r)) \equiv r \rightarrow S^0(r) \equiv \omega^0 - (V')^{-1}(r)$ .

**Resource Constraint (RC):**  $k = R[\omega + S^0(r)] = R[\omega + \omega^0 - (V')^{-1}(r)]$ .

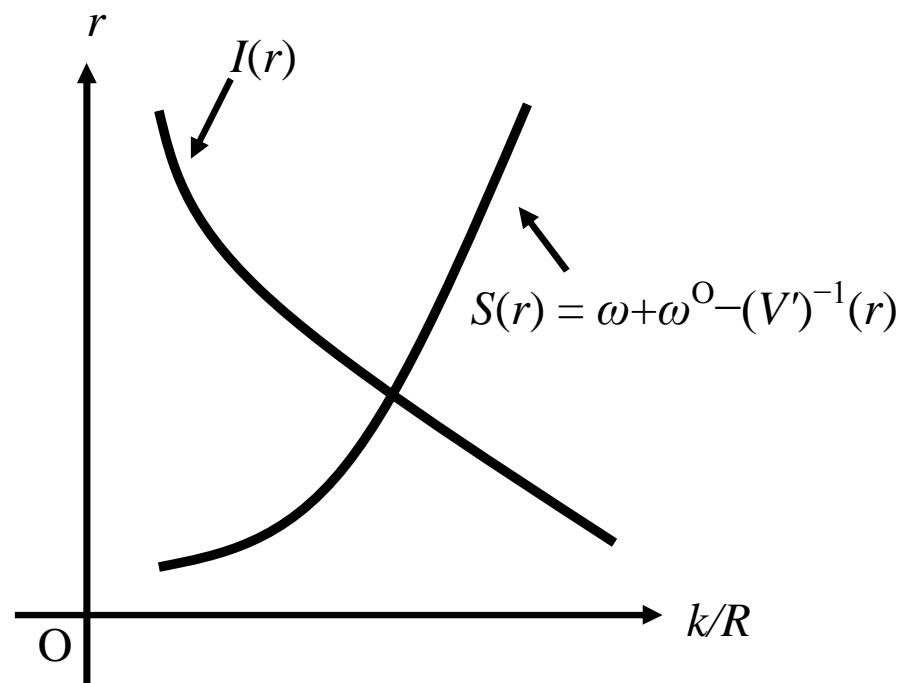
→  $k/R = S(r) \equiv \omega + \omega^0 - (V')^{-1}(r)$ .

**(PC)+ (BC):**  $Rf'(k) = \text{Max}\{1, (1-\omega)/\lambda\}r$ .

→  $k/R = I(r) \equiv \frac{1}{R}(f')^{-1}\left(\text{Max}\left\{1, \frac{1-\omega}{\lambda}\right\} \frac{r}{R}\right)$ .

which jointly determines  $k$  and  $r$ .

- $S(r)$  depends on  $\omega + \omega^0$ ;
- $I(r)$  depends only on  $\omega$ .



Capital Deepening Effect:

$$\Delta\omega^0 > 0$$

Net Worth Effect:

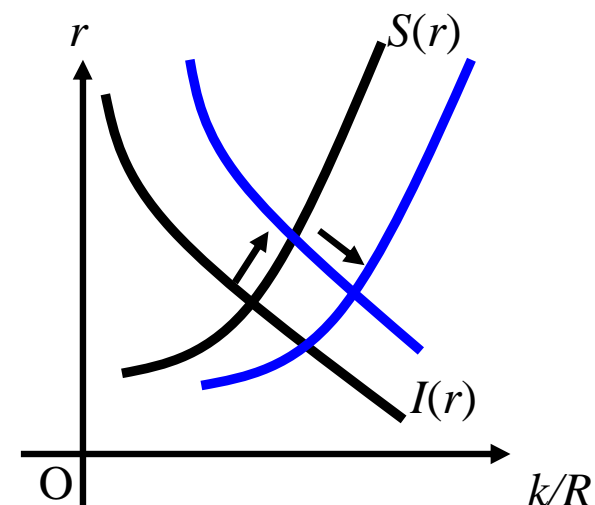
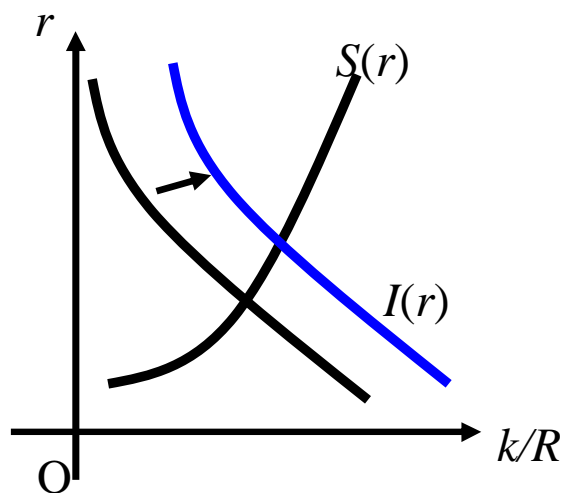
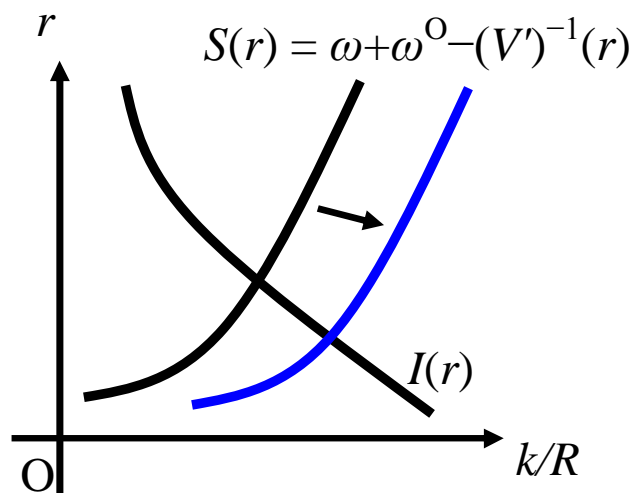
$$\Delta\omega = -\Delta\omega^0 > 0 \text{ (and } \Delta\lambda > 0)$$

when  $\lambda + \omega < 1$ .

Combined Effects:

$$\Delta\omega > 0$$

when  $\lambda + \omega < 1$ .



The equilibrium rate of return is *non-monotonic* in  $\lambda$  (and  $\omega$ );

## A Two-Country Model: Patterns of International Capital Flows

Two Countries: **North and South** of the kind described above

North and South share the same  $f(k)$  and  $R$ , but may differ in  $\lambda$ ,  $\omega$ , and  $\omega^0$ .

Further Assumptions:

- The Input and the Consumption Good are *tradeable*.  $\rightarrow$  This allows the agents to lend and borrow and make the repayment across the borders.
- Physical Capital and the “hidden inputs” is *nontradeable*. We later relax this assumption.
- Only the agents in North (South) can produce Physical Capital in North (South), effectively ruling out FDI. We later relax this assumption.

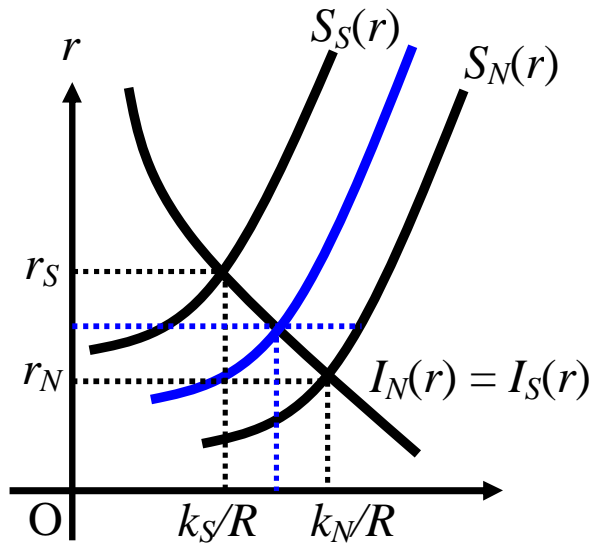
### Experiment:

Suppose the agents in North can pledge  $\varphi\lambda_N$  to the lenders in the South, and the agents in South can pledge  $\varphi\lambda_S$  to the lenders in the North.

Now let  $\varphi$  change from  $\varphi = 0$  (Financial Autarky) to  $\varphi = 1$  (Full Financial Integration).

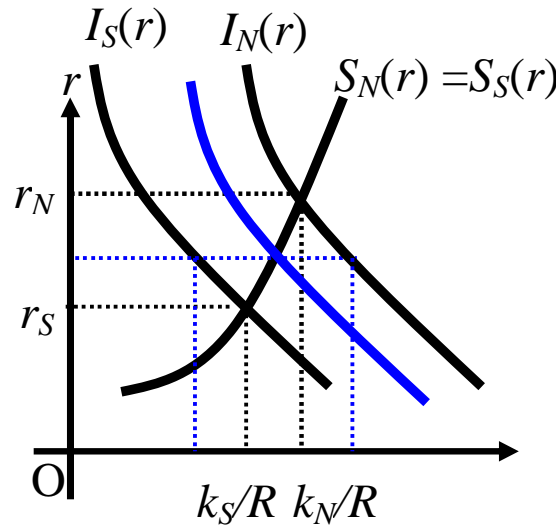
Neoclassical View

$$\lambda_N = \lambda_S, \omega_N = \omega_S, \omega'_N > \omega'_S;$$



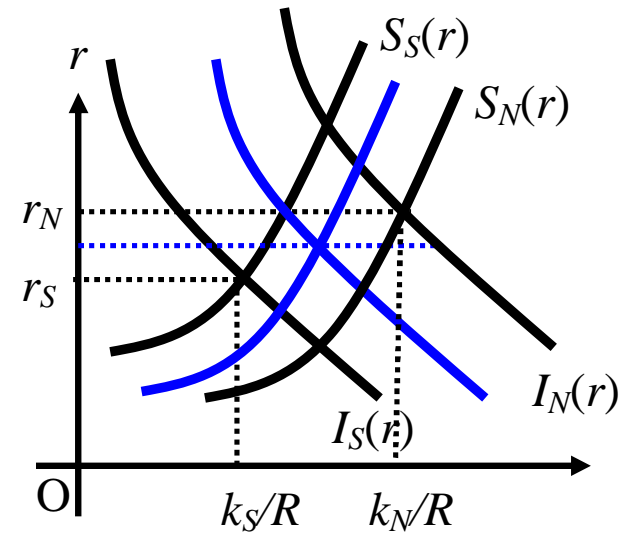
Capital Flight:

$$\lambda_N > \lambda_S, \omega_N = \omega_S, \omega'_N = \omega'_S; \text{ or } \lambda_N = \lambda_S, \omega_N - \omega_S = \omega'_S - \omega'_N > 0.$$



Capital Flight:

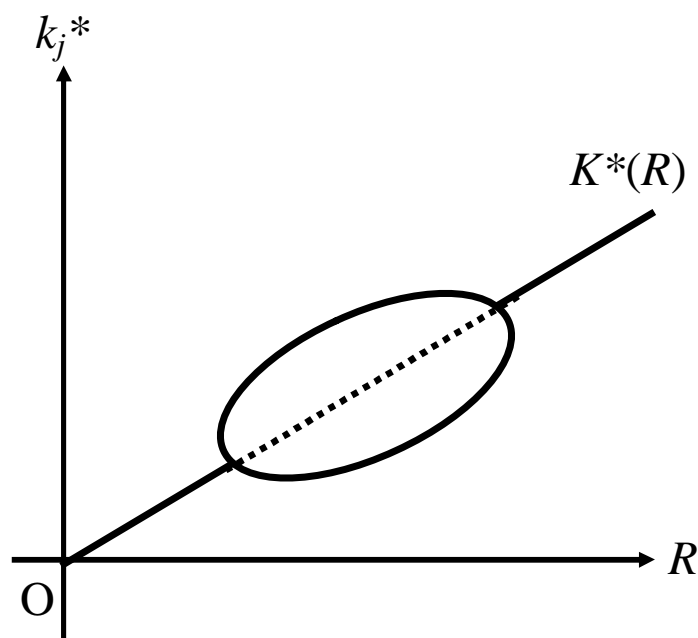
$$\lambda_N = \lambda_S, \omega_N > \omega_S, \omega'_N = \omega'_S.$$



### *Dynamic Implications:*

- Let us introduce a dynamic feedback from  $k_N$  to  $\omega_N$  (and from  $k_S$  to  $\omega_S$ ).
- We can do this by embedding the above structure into an OG framework; so that a higher investment by the current generation leads to a higher demand for the endowment of the next generation, which leads to a higher net worth,  $\omega$ .
- This could lead to Endogenous Inequality across countries from an intermediate value of  $R$ .
- Going from a low value of  $R$  to a higher value of  $R$  could generate Inverted U-curve patterns of Endogenous Inequality.

**Schematically...**



### *Some Other Extensions:*

- Allowing the agents in the North to run the project in the South with reduced productivity could lead to Two-Way Flow of Financial Capital and FDI.
  - Savers in the South lends to Firms in the North, which invest in the South.
  - FDI can be used to bypass the external capital market in the South.
- Introducing Trade in Inputs, which are subject to some trade costs.
  - This could lead to positive spillovers in neighboring countries; Regional contagions (East Asian booms and Latin American stagnations)
- Endogenous Investment Technologies
  - Two-Way Causality between Productivity Differences vs. Credit Market Imperfections
  - Financial Capital may flow into countries with worse credit markets; A solution to the allocation puzzle??



## *General Equilibrium Model with Heterogeneous Projects (with Homogeneous Agents)*

- **A Continuum of Homogeneous Agents with Unit Mass (No Savers)**
- Each Agent can choose one (and only one) of  $J$  non-divisible projects.

	<i>Period 0</i>	<i>Period 1</i>
<i>Type-j Project:</i>	$m_j$ units of the input	$m_j R_j$ units in capital & $m_j B_j$ units in consumption

$m_j$ : the (fixed) set-up cost,  
 $R_j$ : project productivity in capital  
 $B_j$ : project productivity in final good

**Profitability Constraint (PC-j):**  $R_j f'(k) + B_j \geq r$

**Borrowing Constraint (BC-j):**  $m_j [\lambda_j R_j f'(k) + \mu_j B_j] \geq r(m_j - \omega)$ ,

$\lambda_j$ : pledgeability of capital produced by project-j  
 $\mu_j$ : pledgeability of the final good produced by project-j

## Equilibrium Conditions;

$$(1) \quad \omega = \sum_j (m_j n_j).$$

$$(2) \quad k = \sum_j (m_j R_j n_j).$$

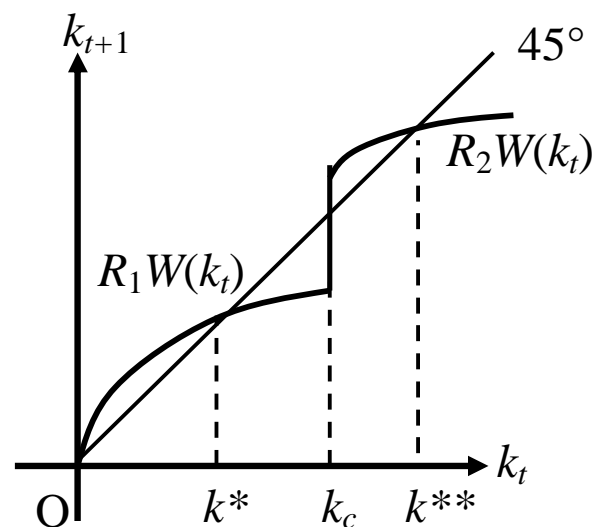
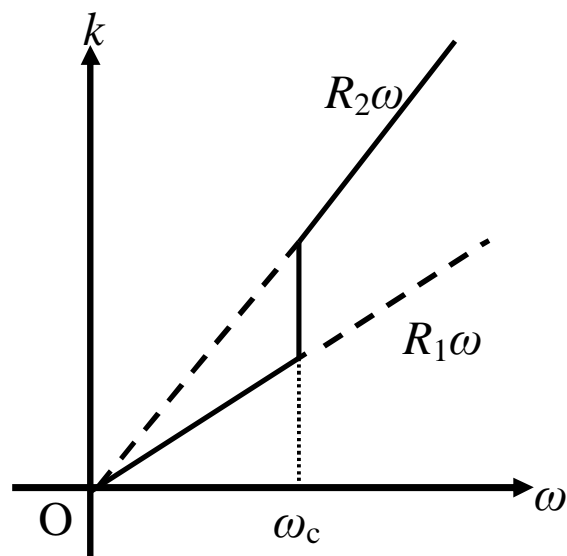
$$(3) \quad r \geq \text{Min} \left\{ \frac{\lambda_j R_j f'(k) + \mu_j B_j}{1 - \omega / m_j}, R_j f'(k) + B_j \right\}; n_j \geq 0 \quad (j = 1, 2, \dots, J)$$

where  $n_j$  is the measure of type- $j$  projects initiated.

Example 1:  $J = 2$ ;  $R_2 > R_1 > \lambda_1 R_1 > \lambda_2 R_2$ .  $B_1 = B_2 = 0$ .

**Key Trade-offs: Productivity vs. Agency Problems;**

Project-2 is more productive, but comes with bigger agency problems than Project-1.

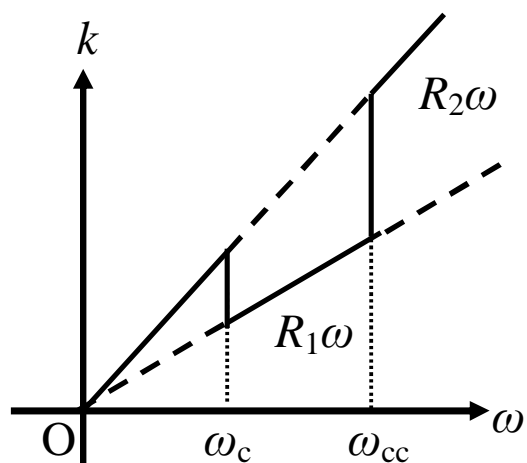


Procyclical Investment Specific Tech Change

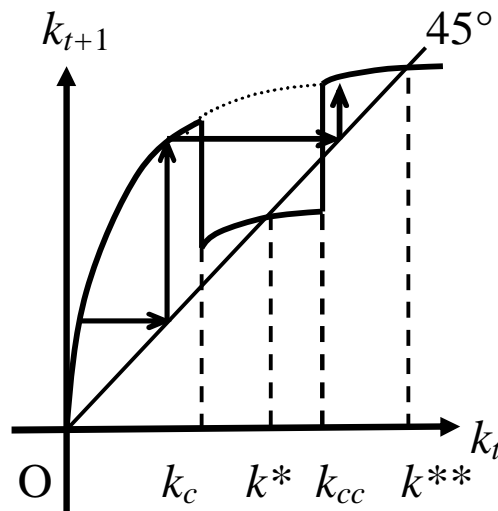
Dynamic Implications: Credit Traps

Example 2:  $J = 2$  and  $R_2 > R_1 > \lambda_2 R_2 > \lambda_1 R_1$ ,  $m_2/m_1 > (1-\lambda_1)/(1-\lambda_2 R_2/R_1) > 1$ .  $B_1 = B_2 = 0$ .

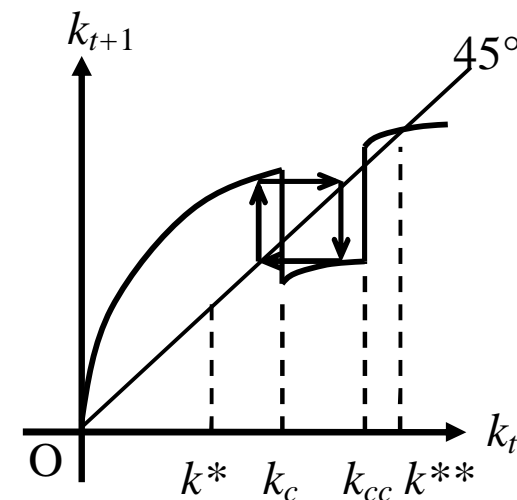
The less productive and less “secure” project-1 have advantage of smaller set-up costs



Counter-cyclical ISTC



Dynamic Implications:  
Leapfrogging

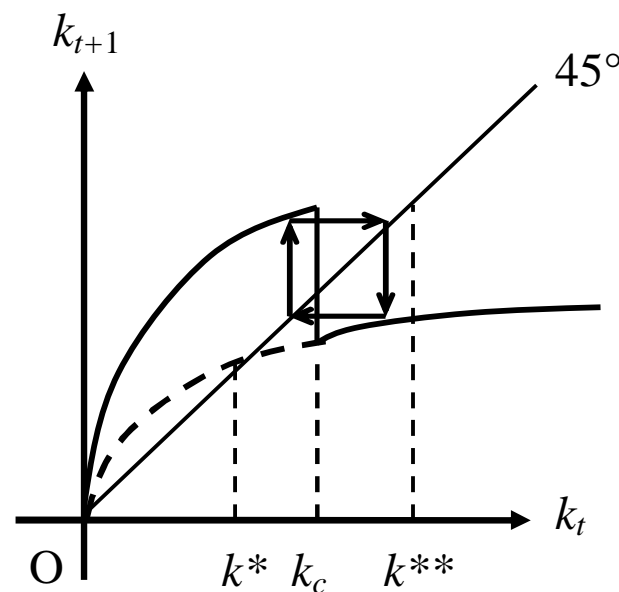
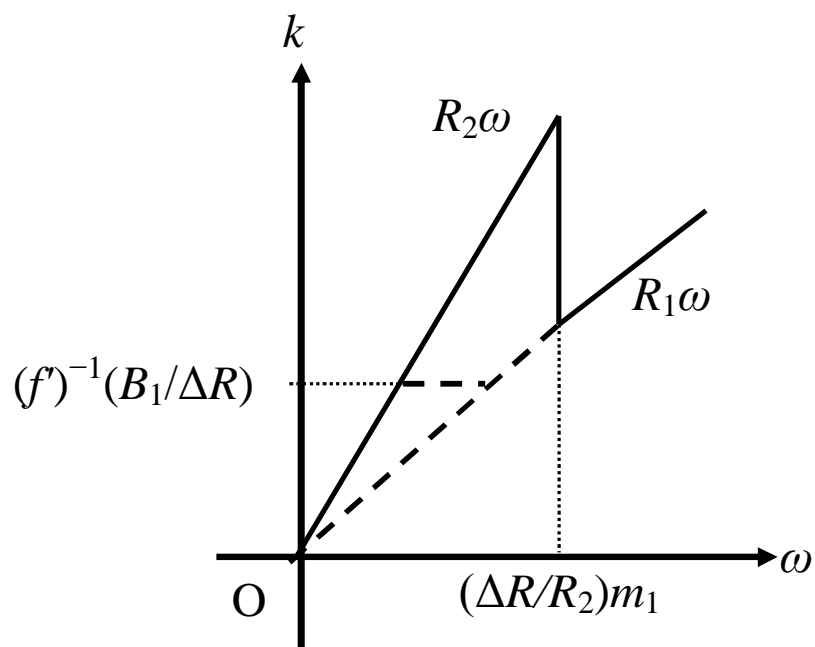


Dynamic Implications:  
Credit Cycles as a Trap

Example 3:  $J = 2$ ;  $\lambda_1 = \lambda_2 = 1$ ,  $\mu_1 = \mu_2 = 0$ ,  $\Delta R \equiv R_2 - R_1 > 0$ ,  $B_1 > B_2 = 0$

Project-1 is less “socially productive” but hgenerates more “private benefits” or “personal satisfaction” than Project-2.

- Project-1 cannot be financed if  $\omega < (\Delta R/R_2)m_1$ .
- If  $B_1 > \Delta R f'(R_1(\Delta R/R_2)m_1)$ , the agents invest to Project-1 whenever  $\omega > (\Delta R/R_2)m_1$ .



- In boom, the entrepreneurs can finance the self-indulgent project.
- In recession, they cannot.

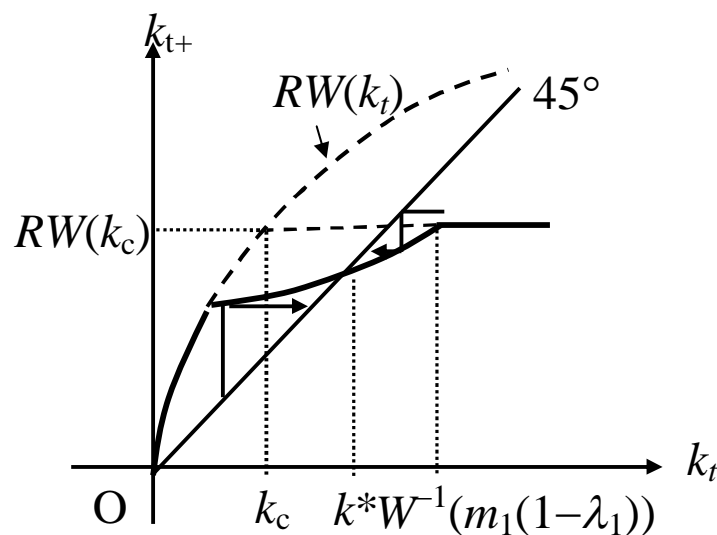
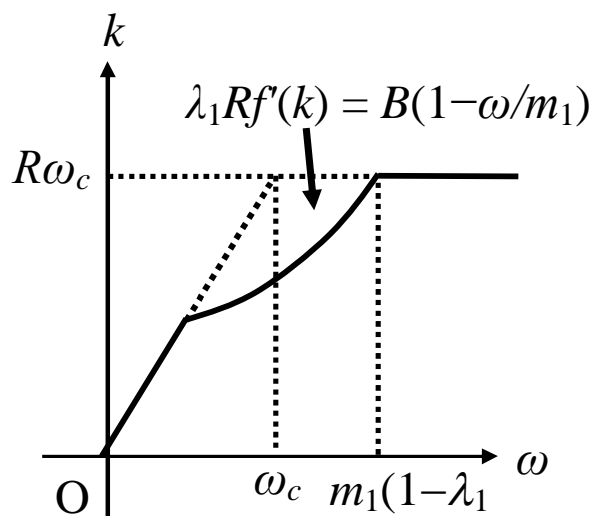
Along these cycles, the booms occur due to the *misallocation* of the credit.

Example 4:  $J = 2$ ;  $R_1 > R_2 = 0$ ,  $B_1 = 0 < B_2$  and  $\lambda_1 < 1$ ,  $\mu_2 = 1$ ,

**Persistence of Inefficient Recessions: Financial Accelerator Models**

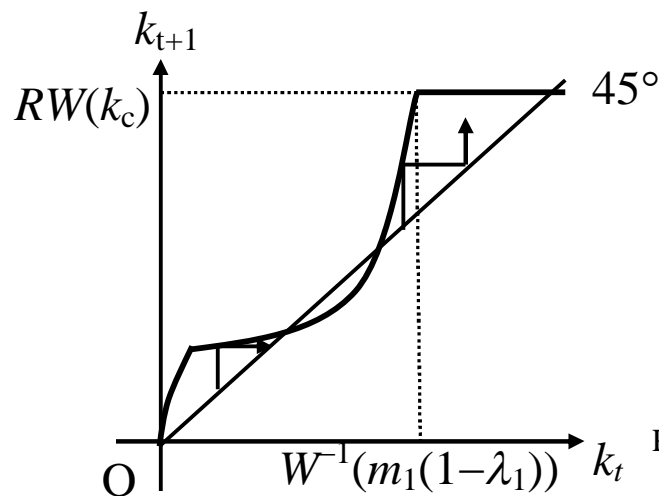
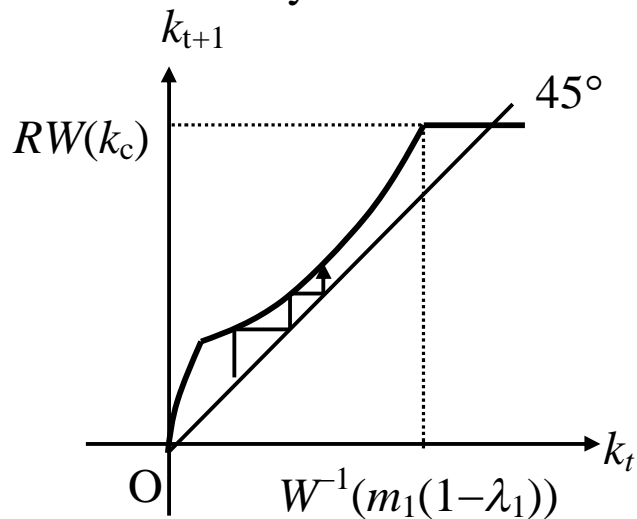
Under-investment of Capital-Generating Project

A Temporary Shock has an Echo Effect



Slow Recovery from Recession

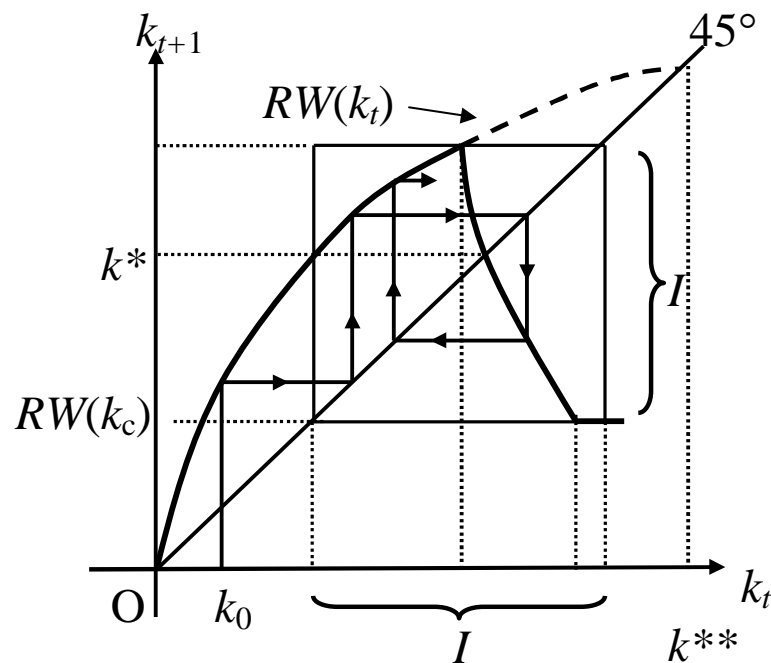
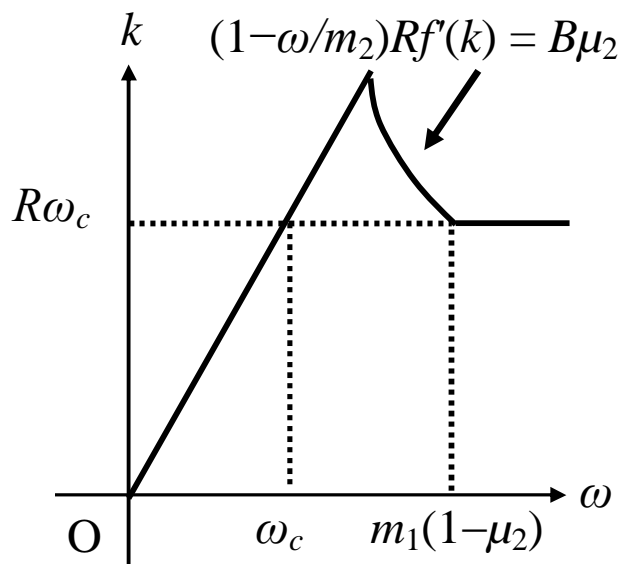
Permanent Recession



Example 5:  $J = 2$ ;  $R_1 > R_2 = 0$ ,  $B_1 = 0 < B_2$  and  $\lambda_1 = 1$ ,  $\mu_2 < 1$ ,  
***Inefficient Booms and Volatility:***

Over-Investment to Capital-Generating Project

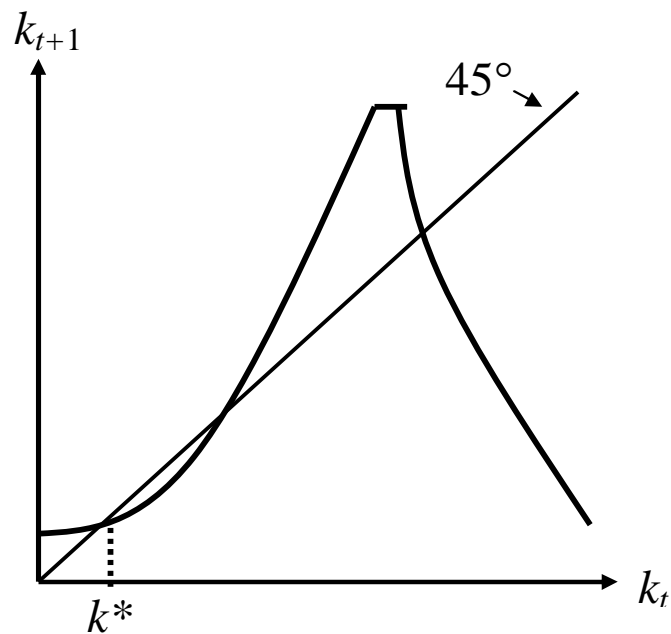
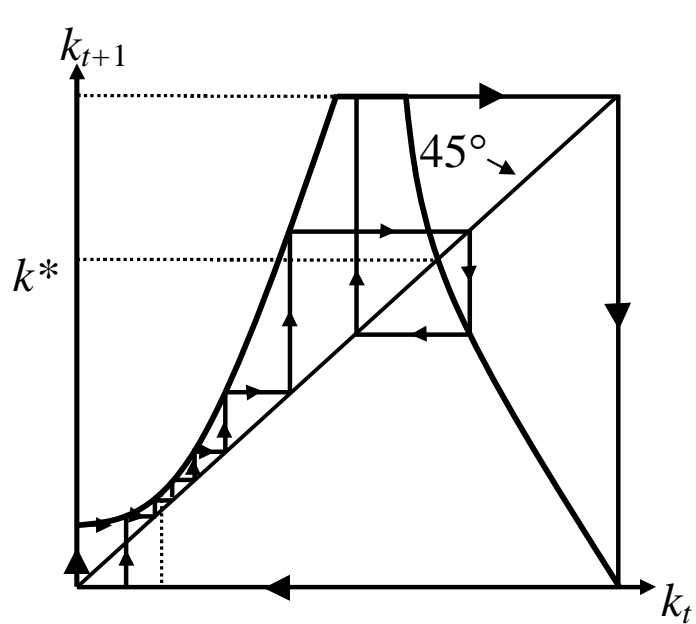
Dynamic Implications:  
 Endogenous Cycles



Again, non-monotonicity; Endogenous Fluctuations Occur for an intermediate value of  $\mu_2$

## Example 6: Hybrid of “Persistence of Inefficient Recessions” & “Inefficient Booms and Volatility” Models

### *Asymmetric Cycles and Intermittent Volatility*





## *A Two-Country Model: Patterns of International Trade:*

Two Countries: **North and South** ( $j = N$  or  $S$ )

**A Continuum of Tradeable Consumption Goods**,  $z \in [0,1]$

Symmetric Cobb-Douglas preferences.

**Homogeneous Agents** with Unit Mass, each endowed with  $\omega < 1$  units of the Input (**Labor**)

**Tradeable Consumption Goods** produced by the projects run by agents

- Each agent can run at most one project.
  - Each project in sector  $z$  converts one unit of labor to  $R$  units of good  $z$ .
- To run the project, one must hire  $1 - \omega$  units of labor at the market wage rate,  $w$ , from those who don't run the project.

**Profitability Constraint (PC- $z$ ):**  $p(z)R \geq w$

**Borrowing Constraint (BC- $z$ ):**  $\lambda \Lambda(z)p(z)R \geq w(1 - \omega)$ ,

$0 \leq \lambda \leq 1$ : country-specific factors

$0 \leq \Lambda(z) \leq 1$ : sector-specific factors, continuous and increasing in  $z$ .

Under  $\omega_N > \omega_S$  and/or  $\lambda_N > \lambda_S$ .

**Autarky Equilibrium:**

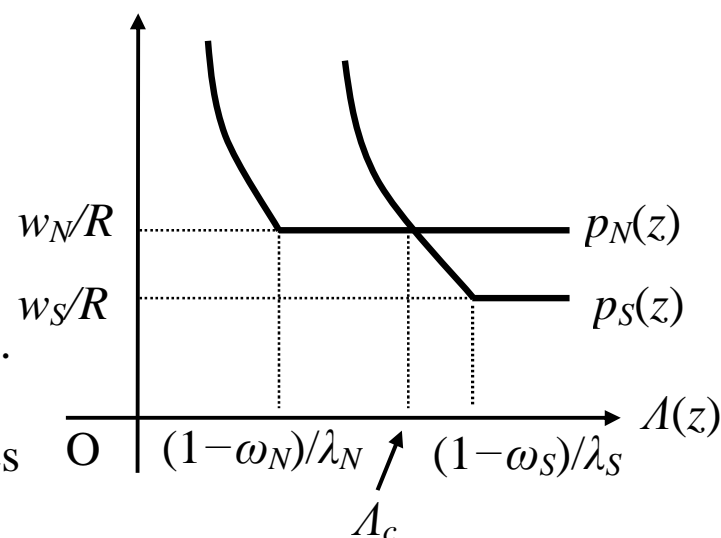
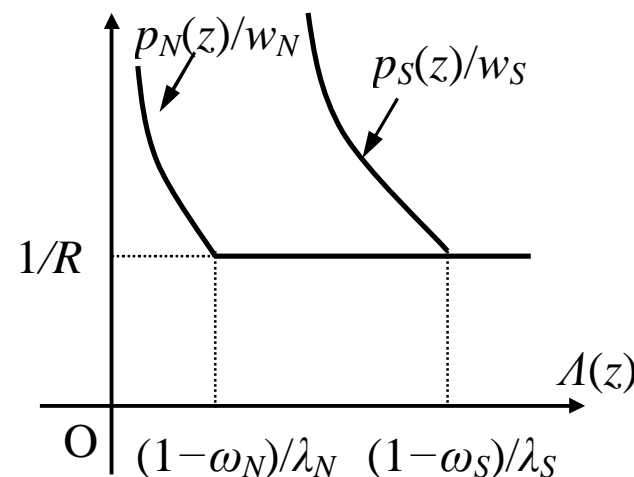
- (PC-z) is binding for  $\Lambda(z) > (1-\omega)/\lambda$ .
  - (BC-z) is binding for  $\Lambda(z) < (1-\omega)/\lambda$ .
- The credit market imperfection restricts entry to the low-indexed sectors.
  - The rent created by the limited entry makes the lenders happy to finance the firms in these sectors.

→ North has *absolute* advantage.

**World Equilibrium:** A higher wage in North.

- North's *comparative* advantage in low-indexed sectors.
- South's *comparative* advantage in high-indexed sectors.

North, with the better contractual environment, specializes in the sectors that are more subject to agency problems.



## *A Model of Polarization:*

**Two Periods:** 0 and 1

### **A Continuum of Agents with Unit Mass:**

- The input endowment at period 0,  $\omega$ , is distributed as  $\omega \sim G(\omega)$ .
- Consumes only at period 1.

### **Two Ways to Convert the Input into Consumption.**

- Can run **an investment project** with **the variable scale**  $I \geq m$ , which converts  $I$  units of the input into  $RI$  units in consumption in period 1, by borrowing  $I - \omega$  at the rate equal to  $r$ . ( $m$  is **the minimum investment requirement**, i.e., investing  $I < m$  generates nothing.)
- **Lending**  $x \leq \omega$  units of the endowment in period 0 for  $rx$  units of consumption in period 1.

### **Agent's Utility = Objective Function = Consumption in Period 1:**

$$\begin{aligned} U &= RI - r(I - \omega) = (R - r)I + r\omega, & \text{if borrow and run the project,} \\ U &= r\omega & \text{if lend (or put in storage).} \end{aligned}$$

If  $r > R$ , the agent does not want to invest.

If  $r = R$ , the agent is indifferent.

If  $r < R$ , the agent *wants to* borrow and invest **as much as possible**.

**Borrowing Constraint:** The agent *can* borrow and invest iff

$$(BC) \quad \lambda R I \geq r(I - \omega).$$

If  $r \leq \lambda R < R$ , the agent could borrow and invest by infinite amount. Never happens in equilibrium!

For  $\lambda R < r < R$ , the agent borrows as much as possible and invest, if it can satisfies the minimum investment requirement.

**Agent's Investment Demand** for  $\lambda R < r < R$ :

$$I(\omega) = \left(1 - \frac{\lambda R}{r}\right)^{-1} \omega \text{ if } \omega \geq m \left(1 - \frac{\lambda R}{r}\right); \quad I(\omega) = 0; \text{ otherwise.}$$

## Credit Market Equilibrium:

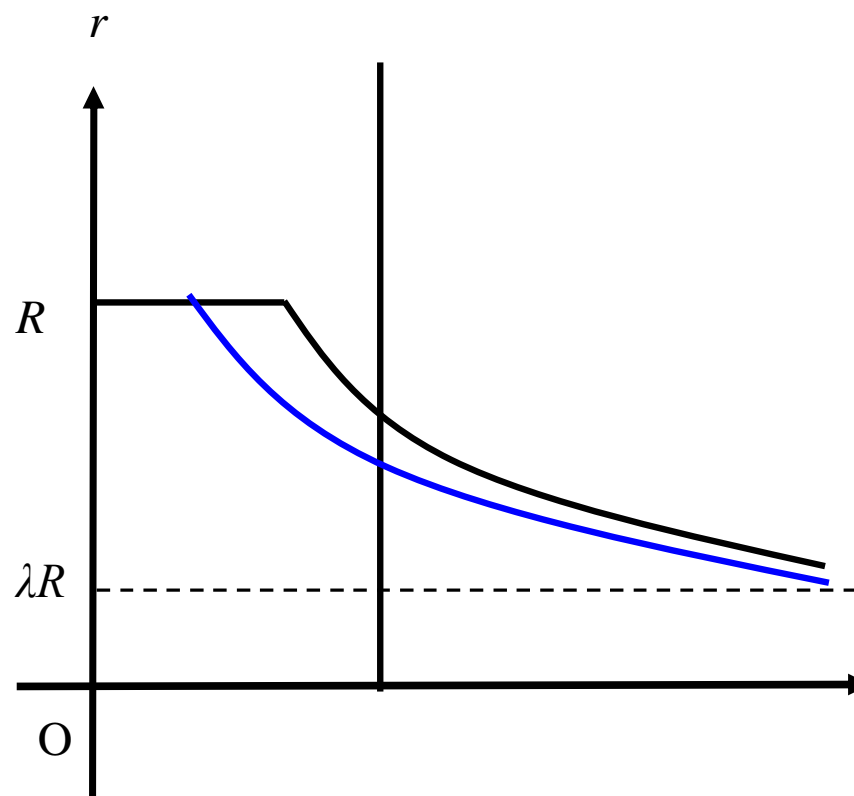
$$\text{Total Supply} = \int_0^\infty \omega dG(\omega) = \left(1 - \frac{\lambda R}{r}\right)^{-1} \int_{m(1-\lambda R/r)}^\infty \omega dG(\omega) = \text{Total Demand}$$

$$\lambda R < r < R$$

$$\text{if } \frac{\int_0^{m(1-\lambda)} \omega dG(\omega)}{\int_0^\infty \omega dG(\omega)} > \lambda.$$

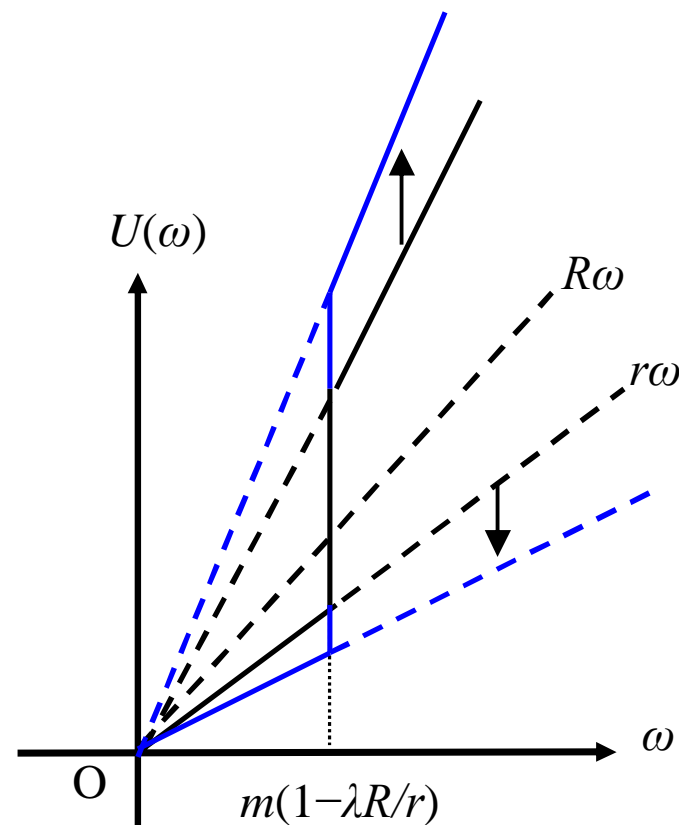
In this range,

a lower  $\lambda$  reduces  $r$ , keeping  $\lambda/r$  constant.



$$U(\omega) = \begin{cases} \frac{1-\lambda}{1-\lambda R/r} R\omega & \text{if } \omega \geq m\left(1-\frac{\lambda R}{r}\right) \\ r\omega & \text{if } \omega < m\left(1-\frac{\lambda R}{r}\right). \end{cases}$$

Note that  $r < R < \frac{1-\lambda}{1-\lambda R/r} R$ .



The marginal value of having an additional unit of the input is strictly

- lower than  $R$  for the poor, unless it would push them above the threshold.
- higher than  $R$  for the rich, because it would enable them to invest more by borrowing more at the market rate strictly lower than the project return  $R$ . (**The Leverage Effect**)

In this model,

- credit market imperfections have no effect on the quantity, or any aggregate variables.
- For any wealth distribution, the *relatively* rich become investors, and the *relatively* poor are prevented from investing.
- A lower  $\lambda$  makes, by reducing  $r$ , enrich the rich who borrow to invest, and impoverish the poor who has no choice but to lend.

→ **A Polarization!** (not necessarily a greater inequality)

*Dynamic Implications:* What if we allow for some feedback from  $U(\omega)$  to  $\omega$ ?

- The Poor may benefit from the credit demand by the rich (Trickle Down Effect)
- Endogenous Inequality

**Interactions between the Rich and the Poor may also take place through Labor Markets.**

A proper discussion of this requires entirely a whole new paper.

## ***Concluding Remarks:***

- Credit Market Imperfections are *rich and diverse* in the aggregate implications.
    - It is so rich that they are useful for understanding a wide range of important issues.
    - It is so diverse that properties of equilibrium often respond *non-monotonically* to parameter changes, suggesting some cautions for studying the aggregate implications of within a narrow class or a particular family of models
  
  - Although this paper synthesizes a diverse set of results with a unified framework, it is far from comprehensive. A large number of issues have not been discussed.
    - Multi-stage financing and liquidity implications
    - Net worth revaluation through asset price changes,
    - Endogenous net worth accumulation by borrowers
    - Endogenous growth, financial intermediation, development of financial markets
    - Asset pricing and monetary policy implications
    - Political economy implications
    - Interacting with other sources of inefficiency such as product market imperfections
- This is merely the tip of the iceberg: more work needs to be done.